Zartash's Notes

First Year Physics

Chapter No 4

Work and Energy

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Questions from Chapter

Q.No.1: **In physics, how many things are involved by the term work?**

Ans: In physics the term work involves two things

(i) Force

(ii) Displacement.

Beside these two things, angle between force and displacement is also involved by the work

Q.No.2: **Define work and also write down its units**

Ans:

Definition:

The work is done by the body if force is applied on the body and it covers some displacement in the direction of force

Mathematical definitions:

The product of magnitude of the force and component of displacement in the direction of the force

In other words, dot product of force and displacement is also called work.

Formula: It is denoted as $W = \vec{F}.\vec{d} = Fdcos\theta$

Units: Its units are Nm or Joule.

Q.No.3: **From the definition of work, what kind of information you get?**

Ans: From the definition of work we find that:

(i) Work is a scalar quantity.

(ii) If $\theta<90^o$, work is done and it is said to be positive

(iii) If $\theta=90^o$, no work is done.

(iv) If $\theta>90^o$, the work is done and said to be negative

(v) If $\theta=0^o$, maximum work is done

(vi) SI unit of work is Nm or joule(J)

Can you derive the value of work done?

i. On the pail when a person holding the pail by force moving in forward direction?

ii. On the wall?

iii. By the centripetal force?

Q.No.4: How much work is done by centripetal force in rotating an object in circular path?

Ans: No work is done by centripetal force in rotating an object in circular path because in that case displacement becomes zero so work will be zero.

As $W = Fd = F(0) = 0$

Q.No.5: Define gravitational field

Ans:

Definition: The space around the earth at which it's gravitational force acts on the body is called gravitational field

Q.No.6: Define conservative field

Ans:

Definition: The field in which the work done by the body is independent of the path followed is called a conservative field.

Examples: Gravitational and electric fields are examples of conservative fields

Q.No.7: Mention the essential conditions of conservative field?

Ans: There are two essential conditions for conservative field.

i) Work done by the body in a conservative field is independent of the path followed

ii) Work done by the body in a conservative field around a close path is zero.

Q.No.8: What is non-conservative force? Give an example of it

Ans:

Definition: The force which depends upon the path followed is called non-conservative force.

Example: The frictional force is non-conservative force because if an object is moved over a rough surface between two points along different paths, the work done against the frictional force depends upon the path followed.

Q.No.9: Define the term power?

Ans:

Definition: Rate of doing work with time is called power. In other words, work done per unit time is named as power

Formula: It is denoted as $P = \frac{W}{t}$

Q.No.10: Define the term watt

Ans:

Definition: If one joule work is done in one second then power will be one watt

Formula: It is denoted as $1\, watt = \frac{1\, joule}{1\, second}$

Units: Its units are watt

Q.No.11: Determine the relationship between power and velocity?

OR Prove that P=F.v ?

Ans: Formula of power is given as

$$\mathrm{P} = \lim_{\Delta t \to o} \frac{\Delta w}{\Delta t}$$

We know that $\Delta W = F.\Delta d$

$$\mathrm{P} = \lim_{\Delta t \to o} \frac{\boldsymbol{F}.\Delta \boldsymbol{d}}{\Delta t}$$

$$P = \boldsymbol{F}. \lim_{\Delta t \to o} \frac{\Delta \boldsymbol{d}}{\Delta t}$$

$$P = \boldsymbol{F}.\boldsymbol{v}$$

Q.No.12: Prove that 1kWh=3.6 M J

Ans:

Definition:

One kilowatt hour is the work done or energy used in one hour by agency whose power is one kilowatt

Proof:

1kWh=1000W×3600s

1kWh =3.6×10^{6}Ws

1kWh =3.6×10^{6}J

1kWh=3.6 M J

Q.No.13: **Define Energy. Also mention its types**

Ans:

Energy: The capacity of a body to do work is called energy. There are basic two forms of energy.

(i) Kinetic Energy

(ii) Potential Energy.

Kinetic energy:

Definition: The energy produced due to the motion of the body is called kinetic energy.

Formula: It is given by the formula $K.E = \frac{1}{2}mv^2$

Potential energy:

The energy produced by a body due to its position is called potential energy

Formula: It is denoted by $P.E = mgh$

Q.No.14: Is kinetic energy is a vector quantity?

Ans: No kinetic energy is a scalar quantity because in its formula velocity gives only its magnitude. So kinetic energy is a scalar quantity

Q.No.15: Explain Work-Energy principle

OR Prove that work done on a body equals the change in its K.E

Ans:

Statement: This law states that work done by a body equals to the change in its kinetic energy

According to third equation of motion $2ad = v_f^2 - v_i^2$

Proof:

$$d = \frac{1}{2a}(v_f^2 - v_i^2) \text{----------(i)}$$

We also know second law of motion $F = ma$------------------(2)

Now multiplying eqs (1) & (2) we get $Fd = ma.\frac{1}{2a}(v_f^2 - v_i^2)$

$$Fd = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

This implies that $W = \Delta K.E.$

Q.No.16: Differentiate between gravitational potential energy and elastic potential energy?

Ans:

Gravitational potential energy	Elastic potential energy
The potential energy due to the gravitational field with respect to the Earth's surface is called gravitational potential energy	The potential energy stored in a compressed spring due to its compression or stretching is called elastic potential energy.
Its formula is $P.E = mgh$	Its formula is $P.E = \frac{1}{2}kx^2$

Q.No.17: Define absolute gravitational potential energy

Ans:

Definition: The work done on a body in moving it from a point to infinity where gravity becomes zero is called the absolute Gravitational Potential Energy.

Formula: It is represented as $U = -\frac{GMm}{r}$

Q.No.18: What is escape velocity? Calculate its value

Ans:

Definition: The velocity of a body by which it escapes out from effect of gravitational field is called escape velocity

Value: Its value for earth is 11 km/s

Q.No.19: What is law of conservation of energy?

Ans:

Statement: This law states that energy neither be created nor be destroyed. It can be transformed from one kind to other kind but total energy remains constant.

Formula: Mathematically it is represented as $T.E = P.E + K.E = Constant$

Q.No.20: What is relation escape velocity and orbital velocity?

Ans: The escape velocity is given as and orbital velocity is given as

$$V_O = \sqrt{\frac{GM}{R}}$$

$$V_{esc} = \sqrt{\frac{2GM}{R}}$$

$$V_{esc} = \sqrt{\frac{2GM}{R}} = \sqrt{2}\sqrt{\frac{GM}{R}}$$

So $$V_{esc} = 1.4V_O$$

Q.No.21: List the non-conventional energy sources

Ans: Following are non-conventional energy sources

i) Energy from tides
ii) Energy from waves
iii) Solar energy
iv) Energy from biomass
v) Energy from waste products
vi) Geothermal energy

Q.No.22: For what purpose Salter's duck is used? Also name its parts

Ans: Salter's duck is used to produce electricity from waves. It consists of two parts.

i) Duck float ii) Balance float

Q.No.23: What is solar cell? Give its significance (uses)

Ans: Solar cell is a device which converts directly sunlight into electrical energy.

Significance(uses):

- They have long life.
- Having very low running cost
- Used in remote ground based weather stations
- In solar calculators and watches

Q.No.24: What is solar constant?

Ans: On a clear day at noon, the intensity of solar energy reaching the atmosphere of earth is about 1.4 kW/m^2 which is referred as solar constant.

Q.No.25: What are the sources of geothermal energy?

Ans: Geothermal energy is obtained from inside the Earth in the form of hot water or steam. It's some sources are given below:

i) Radioactive decay
ii) Residual Heat of earth
iii) Compression of material

Q.No.26: Mention the most common methods for conversion of biomass into fuel

Ans: The most common methods for conversion of biomass into fuel are:

i) Direct combustion
ii) Fermentation

Q.No.27: What is geyser?

Ans: A phenomenon in which a hot spring discharges steam and hot water is called geyser.

Q.No.28: What is aquifer?

Ans: Aquifer is layer of rock holding water that allows water to percolate through it with pressure.

Exercise Questions

Q.No.4.1: A person holds a bag of groceries while standing still, talking to a friend. A car is stationary with its engine running. From the stand point of work, how are these two situations similar?

Ans: In both situations, work will be zero. Because in these cases both objects do not cover any displacement. So work will be zero.

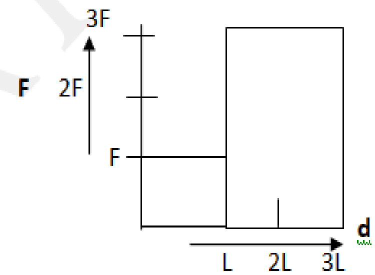
Mathematically: It can be represented as $W = F.d = F(0) = 0$

Q.No 4.2: Calculate the work done in kilo joules in lifting a mass of 10 kg (at steady velocity) through a vertical height of 10 m?

Ans: The formula of work done is given as $W = mgh$

$$W = (10)(9.8)(10)J = 980\,J = 0.98\,kJ$$

Q.No 4.3: A force acts through a distance L. The force is then increased to 3F, and then acts through further distance of 2L. Draw the work diagram to scale



Ans: Net work in this case is given as $W=W_1+W_2$

As $W_1=FL$ and $W_2=6FL$

so $W=FL+6FL=7FL$

Q.No 4.4: In which case is more work done? When a 50 kg bag of books is lifted through 50 cm or when a 50 kg crate is pushed through 2m across the floor with a force of 50 N?

Ans:

First case:	**Second case:**
When $m = 50\,kg$ $h = 50\,cm = 0.5\,m$ Then $W = mgh = (50)(9.8)(0.5)J = 245\,J$	When $F = 50\,N$ and $d = 2\,m$ Then $W = Fd = (50)(2)\,J = 100\,J$
Conclusion: **So in first case more work is done**	

Q.No 4.5: An object has 1 J of potential energy. Explain what does it mean?

Ans: Potential energy has two types. G.P.E and E.P.E. If a body has capacity to do one joule of work during 1 meter height due to gravity then it has 1J of G.P.E and if body has capacity to do one joule of work due to compression and stretching of spring then it has 1J of E.P.E

Q.No 4.6: A ball of mass is held at a height h_1 above a table. The table top is at a height h_2 above the floor. One student says that the ball has potential energy mgh_1 but another student says that it is $mg(h_1+h_2)$. Who is correct?

Ans: Both students are correct. One student take reference point as a surface of table so calculate potential energy mgh_1 and other student take reference point as a surface of earth so calculate potential energy $mg(h_1+h_2)$.

Q.No 4.7: When a rocket re-enters the atmosphere, its nose cone becomes very hot. Where does this heat energy comes from?

Ans: Atmosphere consists of air molecules and dust particles so when a rocket re-enters the atmosphere its nose cone becomes very hot because kinetic energy of rocket changes into heat due to air friction. That's why nose cone of rocket becomes very hot.

Q.No 4.8: What sort of energy in the following:

a) Compressed spring b) Water in a high dam c) A moving car

Ans:

a) In compressed spring elastic potential energy is stored
b) In a high dam, water has gravitational potential energy due to height from earth
c) A moving car has kinetic energy due to its motion

Q.No 4.9: A girl drops a cup from a certain height, which breaks into pieces. What energy changes are involved?

Ans: When a cup has a certain height then it has G.P.E and when it falls down then G.P.E will convert into kinetic energy and on striking the earth, kinetic energy will converted into

- sound energy
- heat energy
- and work done to break the cup into pieces.

Q.No 4.10: A boy uses a catapult to throw a stone which accidently smashes a green house window. List the possible energy changes

Ans: When a stone is in catapult, it has E.P.E. After throwing E.P.E will convert into kinetic energy and on striking the greenhouse window, kinetic energy will be converted into

- sound energy
- heat energy
- and work done to break green house window into pieces.

Long Questions

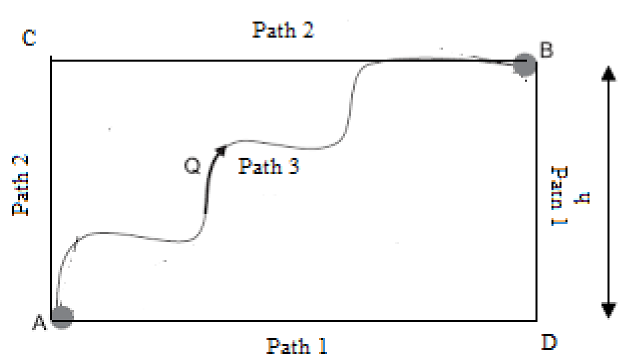
4.3 WORK DONE BY GRAVITATIONAL FIELD

GRAVITATIONAL FIELD:

The space or region around the earth in which its gravitational force acts on the body is called gravitational field

CONSERVATIVE FIELD:

A field in which work done is independent of the path followed is called conservative filed. Or in other words, a field in which works done in a closed path is zero.



GRAVITTAIONAL FIELD IS CONSERVATIVE FIELD:

Earth's gravitational field is conservative field and gravitational force is conservative force. When the object moved in the gravitational field, the work is done by the gravitational force. If the displacement is in the direction of the gravitational force then work done is positive and if the displacement is against the gravitational force, the work is negative

WORK DONE BY GRAVITTAIONAL FIELD:

Let us consider an object of mass m moved with constant velocity v from point A to B along various paths in the presence of the gravitational force as shown in figure. In this case the gravitational force is equal to the m**g** of the object. We take three paths ADB, ACB and AB. We calculate work done along all three paths and will prove that work done along each path will be equal.

WORK DONE ALONG PATH ADB:

The work done by the gravitational field along the path ADB can be split into two parts. The work done along AD and work done along DB. The work done ADB is represented as

$$W_{ADB} = W_{AD} + W_{DB} \quad \text{-----(i)}$$

As the work is given as

$$W = Fd\, cos\theta$$

So work done along AD is

$$W_{AD} = mhg\ Cos90^o$$
$$W_{AD} = 0$$

And work done along DB is

$$W_{DB} = mhg\ Cos180^o$$
$$W_{DB} = -mgh$$

By putting values of work done along AD and BD in equation (i) we get

$$W_{ADB} = 0 + (-mgh)$$
$$W_{ADB} = -mgh$$

WORK DONE ALONG PATH ACB:

The work done by the gravitational field along the path ACB can be split into two parts. The work done along AC and work done along CB. The work done ACB is represented as

$$W_{ACB} = W_{AC} + W_{CB} \quad \text{-----(i)}$$

As the work is given as

$$W = Fd\, cos\theta$$

So work done along CB is

$$W_{CB} = mhg\ Cos90^o$$
$$W_{CB} = 0$$

And work done along AC is

$$W_{AC} = mhg\ Cos180^o$$
$$W_{AC} = -mgh$$

By putting values of work done along AC and CB in equation (i) we get

$$W_{ACB} = (-mgh) + 0$$
$$W_{ACB} = -mgh$$

WORK DONE ALONG PATH AB:

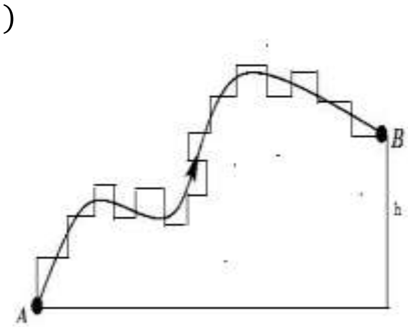
Let us consider now path AB which is curved one. Imagine the curved path is divided into horizontal and vertical steps. There is no work done along the horizontal components because mg is perpendicular to the displacement for these steps. Work done by the force of gravity only along the vertical displacements

$$W_{AB} = W_1 + W_2 + W_3 + \ \ldots\ldots\ldots..+W_N$$
$$W_{AB} = mg\Delta y_1\, cos180^o + mg\Delta y_2\, cos180^o + mg\Delta y_2\, cos180^o + \ \ldots\ldots\ldots..+mg\Delta y_N\, cos180^o)$$
$$W_{AB} = -mg(\Delta y_1 + \Delta y_2 + \Delta y_2 + \ \ldots\ldots\ldots..+\Delta y_N)$$

As $\qquad h = (\Delta y_1 + \Delta y_2 + \Delta y_2 + \ \ldots\ldots\ldots..+\Delta y_N)$

Hence $\qquad W_{AB} = -mgh$

The net amount of work done along AB path is still (-mgh)



CONCLUSION:

Hence it is concluded that work done in the gravitational field is independent of the path followed.

Hence the field in which work done is independent of the path followed or the work done in a closed path be zero is called conservative field.

Gravitational and electric forces are conservative forces and frictional and air resistance are non conservative forces

4.3 ABSOLUTE POTENTIAL ENERGY

DEFINITION:

The absolute potential energy is the value of work done by gravitational force in displacing the object from a given position to infinite position where gravity becomes zero

FORMULA:

The relation for absolute potential energy is given as

$$U = -\frac{GMm}{r}$$

DERIVATION:

The mostly used relation for calculating the work done is mgh. But it is only true when the object is near to the surface of the earth where the gravitational force is nearly constant. If the body is displaced through large distances then gravitational force will not remain constant.

In order to overcome this difficulty, we divide the distance between points 1 and N into small steps each of length Δr so that value of the force remains constant for each step. Hence the total work done can be calculated by adding the work done during all the steps as given

$$W_{1\to N} = W_{1\to 2} + W_{2\to 3} + W_{3\to 4} + - - - - - - - - - - - - + W_{N-1\to N} \quad \text{---------(i)}$$

First of all have to calculate the work done during first step as

$$W_{1\to 2} = F\Delta r \cos 180^o$$

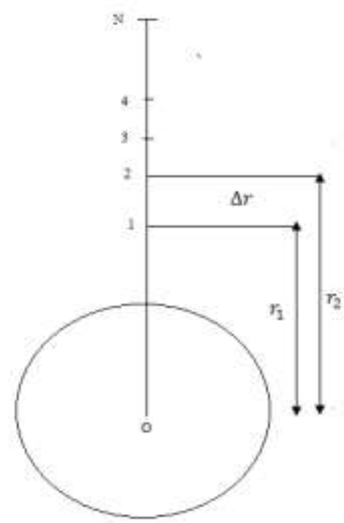
$$W_{1\to 2} = -F\Delta r \quad \text{------------------(ii)}$$

Where F is the gravitational force and is given as

$$F = \frac{GMm}{r^2}$$

For calculating the r^2 we take r_1 and r_2 distances of points 1 and 2 from the centre of the earth respectively.

The distance between centre of the this step and centre of earth is given is



$$r = \frac{r_1 + r_2}{2}$$

From the figure we get

$$r_2 = r_1 + \Delta r$$

Then

$$r_2 - r_1 = \Delta r$$

So

$$r = \frac{r_1 + r_1 + \Delta r}{2}$$

Hence

$$r = \frac{2r_1 + \Delta r}{2} = \frac{2r_1}{2} + \frac{\Delta r}{2} = r_1 + \frac{\Delta r}{2}$$

By squaring on both sides

$$r^2 = \left(r_1 + \frac{\Delta r}{2}\right)^2$$

Hence

$$r^2 = {r_1}^2 + 2(r_1)\left(\frac{\Delta r}{2}\right) + \left(\frac{\Delta r}{2}\right)^2$$

As $(\Delta r)^2 << (r_1)^2$ Hence it can be neglected as compared to $(r_1)^2$

Hence

$$r^2 = {r_1}^2 + r_1 \Delta r$$

By putting the value of Δr we get

$$r^2 = {r_1}^2 + r_1(r_2 - r_1)$$
$$r^2 = {r_1}^2 + r_1 r_2 - {r_1}^2$$
$$r^2 = r_1 r_2$$

By putting the value of r^2 in above mentioned formula of force we get

$$F = \frac{GMm}{r_1 r_2}$$

Now by putting the value of force and Δr in equation (ii)

$$W_{1\to 2} = -\frac{GMm}{r_1 r_2}(r_2 - r_1)$$

Or

$$W_{1\to 2} = -GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

Similarly work done for the second step is given as

$$W_{2\to 3} = -GMm\left[\frac{1}{r_2} - \frac{1}{r_3}\right]$$

And the work done in the last step is

$$W_{N-1\to N} = -GMm\left[\frac{1}{r_{N-1}} - \frac{1}{r_N}\right]$$

By putting all these works in equation(i) we get the total work done from 1 point to N point

$$W_{1\to N} = -GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right] - GMm\left[\frac{1}{r_2} - \frac{1}{r_3}\right] - - - - - - - - - GMm\left[\frac{1}{r_{N-1}} - \frac{1}{r_N}\right]$$

$$W_{1\to N} = -GMm\left[\left[\frac{1}{r_1} - \frac{1}{r_2}\right] + \left[\frac{1}{r_2} - \frac{1}{r_3}\right] - - - - - - - - - \left[\frac{1}{r_{N-1}} - \frac{1}{r_N}\right]\right]$$

On simplification we get

$$W_{total} = -GMm\left(\frac{1}{r_1} - \frac{1}{r_N}\right)$$

If the point N is situated at infinite distance from the earth so

$$r_N = \infty \text{ then } \frac{1}{r_N} = \frac{1}{\infty} = 0$$

Hence

$$W_{total} = -\frac{GMm}{r_1}$$

Therefore for general expression for the gravitational potential energy of a body situated at distance r from the centre of the earth is

$$U = -\frac{GMm}{r}$$

This is also known as the absolute potential energy of the a body at distance r from the centre of the earth.

CONCLUSION:

It is concluded from the above discussion that when r increases, U becomes less negative (increases). Its means when we raise a body from the surface of earth, its potential increases. The change in potential energy as we move a body above the surface of earth is positive.
Absolute potential energy at the surface of the earth can be found by putting the r=R as given below

$$U = -\frac{GMm}{R}$$

The negative sign shows that earth's gravitational field for mass m is attractive.
The above expressions give the energy required to take the body out of Earth's gravitational field, where its potential energy with respect to earth is zero.

4.6 INTERCONVERSION OF POTENTIAL ENERGY AND KINETIC ENERGY

KINETIC ENERGY:

The energy possessed by the body due to its motion is called kinetic energy and it is represented by

$$K.E = \frac{1}{2}mv^2$$

POTENTIAL ENERGY:

The energy possessed by the body due to its position is called potential energy and it is represented by

$$P.E = mgh$$

CONVERSION OF ENERGY:

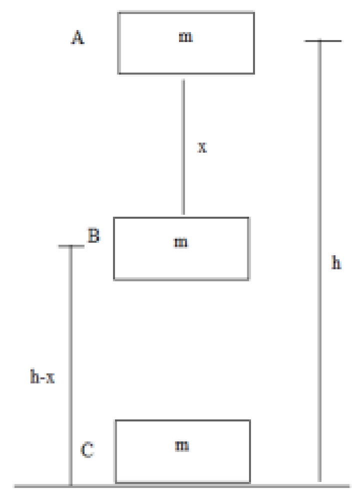
The change in energy from one form to another form is energy conversion. For example when a body falls from certain height, its potential energy converts into kinetic energy.

LAW OF CONSERVATION OF ENERGY:

This law states that energy cannot be created nor be destroyed but can convert from one form to another form but total energy of the system remains constant

$$T.E = K.E + P.E = Constant$$

INTERCONVERSION OF P.E & K.E:



Consider of mass m at rest at a height above the surface of earth as shown in figure. We will calculate the total energy at points A, B & C and will verify that during the falling of this body, potential energy changes into kinetic energy but total energy at all three points will remain constant

TOTAL ENERGY AT POINT A:

At position A the body is at rest so it has K.E=0 and P.E=mgh. Hence its total energy can be calculated as
Its kinetic energy is

$$K.E = \frac{1}{2}mv^2$$
$$K.E = \frac{1}{2}m(0)^2 = 0$$

Its potential energy is

$$P.E = mgh$$

Hence the total energy is

$$T.E = K.E + P.E$$
$$T.E = 0 + mgh$$
$$T.E = mgh$$

TOTAL ENERGY AT POINT B:

At position B the body falls through distance x ignoring the air friction. Hence its total energy can be calculated as

Its kinetic energy is

$$K.E = \frac{1}{2}mv_B{}^2$$

Third equation of motion is given as

$$2aS = v_f^2 - v_i^2$$

By applying conditions of point B on it we get

$$2gx = v_B^2 - v_A^2$$
$$2gx = v_B^2 - (0)^2$$

Hence $$2gx = v_B^2$$

By putting the value of velocity in above equation of kinetic energy we get

$$K.E = \frac{1}{2}m(2gx)$$

So

$$K.E = mgx$$

Its potential energy is

$$P.E = mg(h - x)$$

Hence the total energy is

$$T.E = K.E + P.E$$
$$T.E = mgx + mg(h\text{-x})$$

Or $$T.E = mgx + mgh - mgx$$

So total energy is

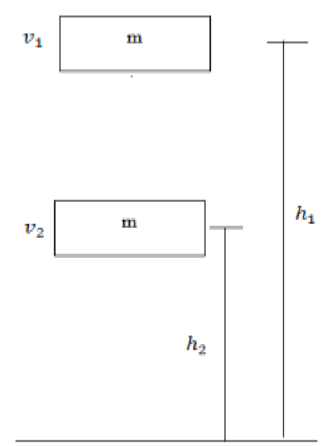
$$T.E = mgh$$

TOTAL ENERGY AT POINT C:

At position C the body falls through distance h ignoring the air friction. Hence its total energy can be calculated as

Its kinetic energy is

$K.E = \frac{1}{2}mv_C{}^2$



Third equation of motion is given as

$$2aS = v_f^2 - v_i^2$$
$$2gh = v_C^2 - v_A^2$$

By applying conditions of point C on it we get

$$2gh = v_C^2 - v_A^2$$
$$2gh = v_C^2 - (0)^2$$

Hence $$2gh = v_C^2$$

By putting the value of velocity in above equation of kinetic energy we get

$$K.E = \frac{1}{2}m(2gh)$$

So

$$K.E = mgh$$

Its potential energy is

$$P.E = mg(0) = 0$$

Hence the total energy is

$$T.E = K.E + P.E$$
$$T.E = mgh + 0$$

So total energy is

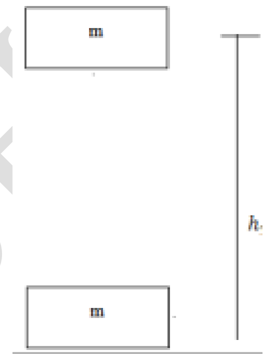
$$T.E = mgh$$

Thus at point C, kinetic energy is equal to the original value of the potential energy of the body. Hence the total energy at all three points remain same.

LOSS AND GAIN OF ENERGIES:

Actually when a body falls, its height decreases. Hence its potential energy also decreases. In other words, when a body falls, its velocity increases due to action of gravity. Hence its kinetic energy increases. Its mean when a body falls, there is loss in its potential energy and gain in kinetic energy which is given as

$$Loss\ in\ P.E = Gain\ in\ K.E$$



According to given figure we get

$$mg(h_1 - h_2) = \frac{1}{2}m({v_2}^2 - {v_1}^2)$$

Where v_1 and v_2 are the velocities at heights h_1 and h_2 respectively. This result is true only when friction force is not considered.

If we assume that friction force is present during the downward motion then apart of potential is used doing the work against the friction equal to fh. Thus this interconvesrion is represented as

Loss in P.E=Gain in KE + Work done against friction

Hence $$mgh = \frac{1}{2}mv^2 + fh$$

CONCLUSION:

It is concluded that when a body falls form a certain height, there will be loss in its potential energy and gain in its kinetic energy but total energy of the system remains constant if air friction is neglected

NUMERICALS

Numerical 4.1: A man pushes a lawn mower with a 40 N force directed at angle of 20^o downward from the horizontal. Find the work done by the man as he cuts a strip of grass 20 m long.

Solution:

$F = 40N$ $\theta = 20^o$ $d = 20\ m$

$W = ?$

$W = Fd\ cos\theta$

$W = (40)(20)\ cos20^o$

$W = 751.7\ J$

$W = 7.5 \times 10^2\ J$

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Numerical 4.2: A rain drop ($m=3.35\times10^{-5}$ kg) falls vertically at a constant speed under the influence of the force of gravity and friction. In falling through 100 m, how much work is done by the (a) gravity and (b) friction

Solution:

$m = 3.35 \times 10^{-5} kg$ $h = 100\ m$

(a) $W = ?$ (due to gravity)

$W = mgh$

$W = \left(3.35 \times 10^{-5}\right)(9.8)(100)J$

$W = 0.0328\ J$

(b) $W = ?$ (due to friction)

$W = -mgh$

$W = -\left(3.35 \times 10^{-5}\right)(9.8)(100)J$

$W = -0.0328\ J$

=======================================

Numerical 4.3: Ten bricks, each 6.0 cm thick and mass 1.5 kg, lie flat on a table. How much work is required to stack them one on the top of the other.

Solution:

$m = 1.5kg$ $h = 6.0\ \ cm = 0.06\ m$

$W = ?$

$W = mgh$

Total work for ten bricks

$W = 45mgh$

$W = 45(1.5)(9.8)(0.06)J$

$W = 39.69\ J = 40\ J$

=======================================

Numerical 4.4: A car of mass 800 kg travelling at 54 km/h is brought to rest in 60 meters. Find the average retarding force on the car. What has happened to the original kinetic energy?

Solution:

$m = 800\ kg$ $v_i = 54\frac{km}{h} = 54\frac{1000\ m}{3600\ s} = 15\frac{m}{s}$

$S = 60\ m$ $v_f = 0$

$F = ?$ $K.E = ?$

$$F \times d = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$F \times 60 = \frac{1}{2}(800)(0)^2 - \frac{1}{2}(800)(15)^2\ \text{N}$$

$$F \times 60 = (0 - 90000)\ \text{N}$$

$$F = \frac{-90000}{60}N$$

$$F = -1500\ N$$

$$-F = 1500\ N$$

As the car is brought to be rest so its original kinetic energy will be zero

$$K.E = 0$$

Numerical 4.5: A 1000 kg automobile at the top of incline 10 meter high and 100 meter long is released and rolls down the hill. What is the speed at the bottom of the incline if the average retarding force due to friction is 480 N?

Solution:

$m = 1000\ kg$ $h = 10\ m$ $S = 100m$

$f = 480\ N$ $v = ?$

$$mgh = \frac{1}{2}mv^2 + fS$$

$$(1000)(9.8)(10) = \frac{1}{2}(1000)v^2 + (480)(100)$$

$$98000 = (500)v^2 + (48000)$$

$$98000 - 48000 = (500)v^2$$

$$50000 = (500)v^2$$

$$v^2 = 100$$

$$\sqrt{v^2} = \sqrt{100}ms^{-1}$$

$$v = 10ms^{-1}$$

Numerical 4.6: 100 m^3 of the water is pumped from a reservoir into tank, 10 m higher than the reservoir, in 20 minutes, if density of the water is 1000 kgm^{-3}, find

(a) The increase in potential energy

(b) The power delivered by the pump

Solution:

$\rho = 1000\frac{kg}{m^3}$ $h = 10\, m$

$V = 100\, m^3$ $t = 20\, minutes = 1200\, s$

$\rho = \frac{m}{V}$

$m = \rho V = (1000)(100)\, kg = 10^5 kg$

(a) $\boldsymbol{P.E} = ?$

$P.E = mgh$

$P.E = (10^5)(9.8)(10)J$

$P.E = 9.8 \times 10^6 J$

(b) $\boldsymbol{P} = ?$

$$P = \frac{W}{t} = \frac{mgh}{t}$$

$$P = \frac{9.8 \times 10^6}{1200} watt$$

$$P = 8166.6\, watt = 8.2\, kW$$

Numerical 4.7: A force (thrust) of 400 N is required to overcome road friction and air resistance in propelling an automobile at 80 km/h. What power (kW) must the engine develop?

Solution:

$F = 400\, N$

$v = 80\, kmh^{-1} = \frac{80\times1000\, m}{3600\, s} = 22.2\, ms^{-1}$

$P = ?$

$P = F.v = (400)(22.2) watt$

$P = 8889\, watt$

$P = \frac{8889}{1000}\, kW$

$P = 8.9\, kW$

Numerical 4.8: How large force is required to accelerate an electron (m=9.1×10^{-31} kg) from rest to a speed of 2.0×10^7 m/s through a distance of 5.0 cm?

Solution:

$F = ?$ $v_i = 0$

$m = 9.1 \times 10^{-31}$ kg $v_f = 2.0 \times 10^7$ m/s

$d = 5cm = \frac{5}{100} m = 0.05\, m$:

$$F \times d = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$F \times (0.05) = \frac{1}{2}(9.1 \times 10^{-31})(2.0 \times 10^7)^2 - \frac{1}{2}(9.1 \times 10^{-31})(0)^2$$

$$F \times (0.05) = 1.82 \times 10^{-16\,\text{N}} - 0$$

$$F = 3.6 \times 10^{-15}\text{N}$$

$$F = 3.6 \times 10^{-15}\text{N}$$

Numerical 4.9: A diver weighing 750 N dives from a board 10 m above the surface of the pool of water. Use the conservation of mechanical energy to find his speed at a point 5.0 m above the water surface, neglecting air friction.

Solution:

$W = 750\, N$ $h_1 = 10\, m$ $h_2 = 5\, m$

$v = ?$

$$\frac{1}{2}mv^2 = mg(h_1 - h_2)$$

$$\frac{1}{2}v^2 = g(h_1 - h_2)$$

$$\frac{1}{2}v^2 = 9.8(10 - 5)$$

$$v^2 = 98$$

$$\sqrt{v^2} = \sqrt{98}\, ms^{-1}$$

$$v = 9.9\, ms^{-1}$$

Numerical 4.10: A child start from rest at the top of a slide of height 4.0 m. (a) What is his speed at the bottom if the slide is frictionless? (b) if he reaches the bottom, with a speed of 6 m/s, what percentage of his total energy at the top of slide is lost as a result of friction?

Solution: $h = 4\, m$

(a) $v = ?$

$$\frac{1}{2}mv^2 = mgh$$

$$\frac{1}{2}v^2 = gh$$

$$\frac{1}{2}v^2 = (9.8)(4)$$

$$v^2 = 78.4$$

$$v = 8.8 ms^{-1}$$

(b) $\%age\ loss\ in\ K.E = ?$

$$v^/ = 6\, ms^{-1}$$

$$K.E = \frac{1}{2}mv^2 = \frac{1}{2}m(8.8)^2 J$$

$$K.E = (38.72\, m)J$$

$$K.E^/ = \frac{1}{2}mv^{/2} = \frac{1}{2}m(6)^2 J$$

$$K.E^/ = (18m)J$$

$$loss\ in\ K.E = (38.72\, m)J - (18m)J$$

$$Loss\ in\ KE = (20.72\, m)J$$

$$\%age\ loss\ in\ K.E = \frac{20.72\, m}{38.72\, m} \times 100\%$$

$$\%age\ loss\ in\ K.E = 54\ \%$$