MATHEMATICS 10th Science Group

7/18/2020

Chapter 6.

BASIC STATISTIC

A project of: www.notespk.com

Contact or Suggest Us: info@notespk.com

Contents

Exercise 6.1

The following data shows the number of members in various families. Construct frequency distribution. Also find cumulative frequencies.

$$9, 11, 4, 5, 6, 8, 3, 4, 9, 12, 8, 9, 10, 6, 7, 7, 11, 4, 4, 8, 4, 3, 2, 7, 9, 10, 9, 7, 6, 9, 5, 7$$

Solution:

Frequency distribution of numbers of family members.

Numbers of members	Talley marks	Frequency	Commutative
2	I	1	1
3	III	3	1+3=4
4	卌I	6	4+6=10
5	IIII	4	10+4=14
6	III	3	14+3=17
7	卌I	6	17+6=23
8	卌	5	23+5=28
9	卌 I	6	28+6=34
10	II	2	34+2=36
11	II	2	36+2=38
12	I	1	38+1=39
Total		39	

Question No.2 the following data has been obtained after weighing 40 students of class V. Make a frequency distribution taking class interval size as 5. Also find the class boundaries and midpoints.
34,26,33,32,24,21,37,40,41,28,31,33,34,37,23,27,31,31,36,29,35,36,37,38,22,27,28,29,31,35,35,40,21,32, 33,27,29,30,23.
Also make a less than cumulative frequency distribution.(Hint: Make classes 20--24,25—29).
Solution:

	Frequency Distribution	
Class limits	Talley marks	Frequency
$20 - 24$	卌	6
$25 - 29$	卌 卌	10
$30 - 34$	卌 卌 I	12
$35 - 39$	卌 IIII	9
$40 - 44$	III	3
Total		40

Cumulative frequency Distribution

Class Boundaries	Frequency f	Cumulative frequency	Class Boundaries	Cumulative frequency
$14.5 - 19.5$	0	0	Less than 19.5	0
$19.5 - 24.5$	6	$0 + 6 = 6$	Less than 24.5	6
$24.5 - 29.5$	10	$6 + 10 = 16$	Less than 29.5	16
$29.5 - 34.5$	13	$16 + 13 = 29$	Less than 34.5	29
$34.5 - 39.5$	8	$29 + 8 = 37$	Less than 39.5	37
$40 - 44$	3	$37 + 3 = 40$	Less than 44.5	40

Question No.3 from the following data representing the salaries of 30 teachers of a school. Make a frequency distribution taking class interval size of $Rs.100$, 450,500,550,580,1020,1130,1220,760,690,710,750,1120,760,1240.(Hint: Make classes $450 - 349, 550 - 649, \dots$).

Solution:

Frequency Distributive Table

Class Limits	Talley marks	Frequency
$450 - 549$	||	2
$550 - 649$	||	2
$650 - 749$	||||	4
$750 - 849$	卌	5
$850 - 949$	|||	3
$950 - 1049$	||||	4
$1050 - 1149$	卌	5
$1150 - 1249$	卌	5
	$Total =$	30

(a) ***Find the most frequent load shedding hours.***

$\mathbf{6 - 7}$

(b) ***Find the least load shedding intervals.***

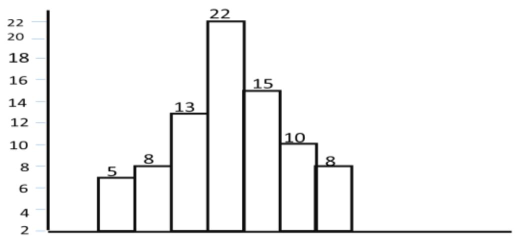
$\mathbf{4 - 5}$

Question No..5 Construct a Histogram and frequency Polygon for the following data showing weights of a studying in kg.

Weights	Frequency / No of students
$20 - 24$	5
$25 - 29$	8
$30 - 34$	13
$35 - 39$	22
$40 - 44$	15
$45 - 49$	10
$50 - 54$	8

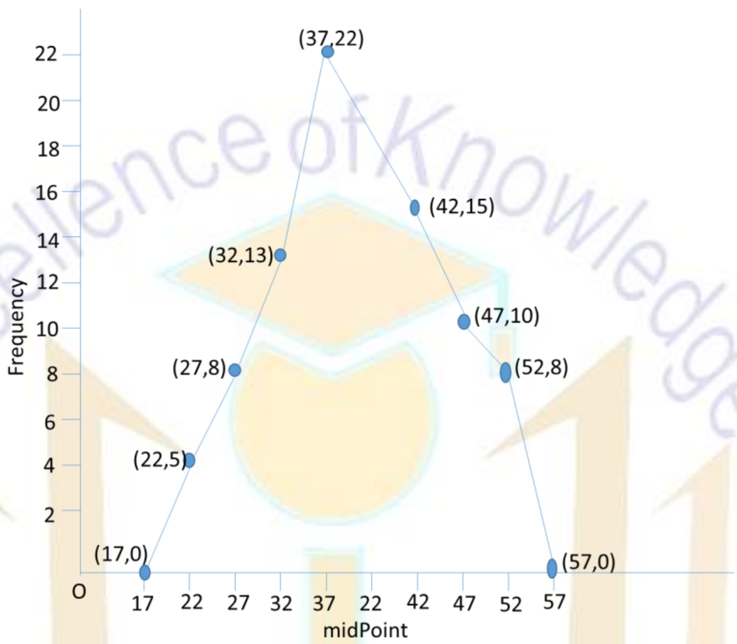
Solution:

Class Boundaries	Frequency / No of students
$19.5 - 24.5$	5
$24.5 - 29.5$	8
$29.5 - 34.5$	13
$34.5 - 39.5$	22
$39.5 - 44.5$	15
$44.5 - 49.5$	10
$49.5 - 54.5$	8



Class Boundaries (Weights)

Class Limits	Mid points	Frequency
$20-24$	22	5
$25-29$	27	8
$30-34$	32	13
$35-39$	37	22
$40-44$	42	15
$45-49$	47	10
$50-54$	52	8



Measures of Central Tendency:

A specific valves of the variable around which the majority of the observation tend to concentrate, this reprehensive shows the tendency or behavior of the distribution of the variable under study. This value is called average or of the central value. The measure or techniques that are used to determine this central value are called Measures of Central Tendency.

The following measures of central tendency will be discussed in this section:

1. Arithmetic Mean
2. Median
3. Mode
4. Geometric mean
5. Harmonic mean
6. Quartiles

Arithmetic Mean:

Arithmetic mean (or simply called mean) is a measure that determines a value (observation) of the variable under study by dividing the sum of all valves(observations) of the variable by their number of observations. We denote Arithmetic mean by $\overline{X}$ in symbols we define:

Arithmetic mean $\overline{X} = \frac{\sum X}{n} = \frac{sum\ of\ all\ valves\ of\ observation}{No.of\ observation}$

Computation of Arithmetic Mean:

There are two types of data, ungrouped and grouped. We, therefore have different methods to determine Mean for the two types of data.

Ungrouped Data:

For ungrouped data we uses three approaches to find mean. These are as follows.

i. **Direct Method(By definition)**
The formula under this method is given by:

$$\bar{X} = \frac{\sum X}{n} = \frac{sum\ of\ all\ valves\ of\ observation}{No.\ of\ observation}$$

ii. **Indirect, Short Cut or Coding Methods:**
There are two approaches under indirect method. There are used to find mean when data set consist of large valves or large number of valves. The purpose is to simplify the computation of Mean. These approaches exist in theory but are not used in practice as many statistical software are available now to handle large data. However a student should have knowledge of these two approaches. These are:

I. Using an Assume or provisional Mean
II. Using a provisional mean and changing scale of the variable.

Deviation is defined as difference of any value of the variable from any constant "A" for Example we say,

Deviation from Mean of $X = (x_1 - \bar{X})\ \ for\ i = 1, 2, \dots n$

Deviation from any constant A= $(x_1 - A)\ \ for\ i = 1, 2, \dots n$

The formulae used under indirect method are:

$$(i) \qquad \bar{X} = A + \frac{\sum_{i=1}^{n} D_1}{n}$$

$$(ii)\ \ \bar{X} = A + \frac{\sum_{i=1}^{n} U_1}{n} \times h$$

Where $D_1 = (x_1 - A), A\ is\ any\ assumed\ value\ of\ X\ called\ Assumed\ or\ provisonal\ mean.$

$$U_1 = \frac{x_1 - A}{h}, h\ is\ a\ constant\ multiple\ of\ the\ valves\ of\ X.$$

Grouped data:

A data in the form of frequency distributive is called grouped data. For the grouped data we define formale under direct and indirect methods are given below.

(a) Using Direct method

$$\bar{X} = \frac{\sum fx}{\sum f}$$

Using indirect method

$(i) \qquad \bar{X} = A + \frac{\sum fD}{\sum f} \quad (ii) \quad \bar{X} = A + \frac{\sum fu}{\sum f} \times h$

Where "X" denotes the midpoint of a class or group if class intervals are given and "h" is the class interval size.

(b) Median:

Median is the middle most observation in an arranged data set. It divides the data set into two equal parts. $\bar{X}\ is\ used$ to represent median. We determine Median by using the following formulae.

Ungrouped data:

Case-I:

When the number of observations is odd of a set of data arranged in order of magnitude the median (middle most observation) is located by the formula given below:

Median $= sizeof\ \left(\frac{n+1}{2}\right)^{th}\ observation$

Case-2:

When the number of observation is even of a set of data arranged in order of magnitude the median is the arithmetic mean of the two middle observation. That is, median is average of

$\frac{n}{2}\ and\ \left(\frac{n}{2}+1\right)^{th}\ Valves.$

Median $= \frac{1}{2}\left[size\ of\ \left(\frac{n}{2}th+\frac{n+1}{2}th\right)observation\right]$

Grouped Data (Discrete)

The following steps are involved in determining median for grouped data (discrete)

i. **Make cumulative frequency column.**
ii. **Determine the median observation using cumulative frequency, i.e the class containing** $\left(\frac{n}{2}\right)^{th}$ **observation.**

Grouped Data (continuous):

The following steps are involved in determining median for grouped data(continuous)

i. **Determine Class boundaries**
ii. **Make cumulative frequency column.**

Determine the median class using cumulative frequency. i.e the class containing $\left(\frac{n}{2}\right)^{th}$ **observation.**

Use the formula:

$$Median = l + \frac{h}{f}\left\{\frac{n}{2} - c\right\}$$

Where $l = lower\ class\ boundary$ **of the median class**

h **=class interval size of the median class**

$f = frequency$ **of the median class,**

c **=cumulative frequency of the class preceding the median class.**

Mode:

Mode id defined as the most frequent occurring observation in the data. It is the observation that occur maximum number of times is given data. The following formula is used to determine mode.

i. **Ungrouped data and Discrete data**
   Mode$=$ $the\ most\ frequent\ observation$
ii. **Grouped Data (continuous)**

The following steps are involved in determine mode for grouped data:

Find the group that has the maximum frequency.

Use the formula

$$Mode = l + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times h$$

Where $l = lower$ class boundary of the model class or group,

$h =$ class interval size of the model class,

$f_m = frequency$ of the model class,

$f_1 = frequency\ of\ the\ class$ preceding the model class.

f_2 =frequency of the class succeeding the model class

Geometric Mean:

Geometric Mean of a variable X is the n^{th} positive root of the product of the $x_1, x_2, x_3, \dots, x_n$ observations. In symbols we write,

$$G.M = (x_1, x_2, x_3, \dots, x_n)^{\frac{1}{n}}$$

The above formula can also be written by using logarithm.

For Ungrouped data

$$G.M = Antilog\left(\frac{\sum logX}{n}\right)$$

For Grouped data

$$G.M = antilog\left(\frac{\sum logX}{\sum f}\right)$$

Harmonic Mean:

Harmonic Mean refers to the valve obtained by reciprocating the mean of the reciprocal of $x_1, x_2, x_3, \dots, x_n$ observation. In symbols, for ungrouped data,

$$H.M = \frac{n}{\sum \frac{1}{x}}$$

Properties of Arithmetic Mean:

i. Mean of a variable with similar observation say constant k is the constant k itself.
ii. Mean is affected by change in origin.
iii. Mean is affected by change in scale.
iv. Sum of the deviations of the variables X from its mean is always zero.

Calculation of Weighted Mean and Moving Average:

The Weighted Arithmetic Mean:

The relative importance of a number is called its weight. When numbers $x_1, x_2, \dots, x_n$ $are\ not\ equally$

Important, we associative them with certain weights$w_1, w_2, w_3, \dots, w_n$

Depending on the impotence or significance.

$$\overline{X} = \frac{w_1x_1 + w_2x_2 + \cdots + x_nx_n}{w_1 + w_2 + \cdots + w_n} = \frac{\sum wx}{\sum w}$$

Is called the weighted arithmetic mean.
Moving Average:
Moving average are defined as the successive average (arithmetic means) which are computed for a sequence of days/ months/ years at a time. If we want to find 3-days moving average. We find the average of first 3-days, then dropping the first day and add the succeeding day to this group. Place the average of each 3-days against the mid of days.

Exercise 6.2

1. What do you understand by measures of central tendency?
Solution:

The specific value of the variable around which the majority of the on observations tend to concentrate is called the central tendency.

2. Define Arithmetic mean, geometric mean, Harmonic mean, mode and Median?
Solution:

i. Arithmetic Means:
Mean is a measure that determine a value of the variable understudy by dividing the Sum of all valves of the variable by their number of observations.

$$\bar{X} = \frac{\sum X}{n} (for\ ungrouped\ data)\ and\ \bar{X} = \frac{\sum fX}{\sum f} (for\ grouped\ data)$$

ii. Geometric Means
Geometric mean of a variable x is the nth positive root of the product of the $x_1, x_2, x_3, \ldots, x_n$ observation. G.M $=(x_1, x_2, x_3, \ldots, x_n)^{\frac{1}{n}}$

iii. Harmonic Means:
Harmonic mean refers to the value obtained by reciprocating the mean of the reciprocal of $x_1, x_2, x_3, \ldots, x_n\ observations.$

$$H.M = \frac{n}{\sum \frac{1}{x}} (for\ ungrouped\ data)\ and\ H.M = \frac{n}{\sum \frac{f}{x}} (for\ grouped\ data)$$

iv. Mode:
The most repeated value in an observation is called mode.

v. Median
Median is the middle most observation in an arranged data set. It divides the data set into equal parts.

3. Find arithmetic mean by direct method for the following set of data:
i. 12,14,17,20,24,29,35,45
ii. 200,225,350,375,270,320,290

Solution:

i. $A.M = \bar{X} = \frac{\sum X}{n} = \frac{12+14+17+20+24+29+35+45}{8}$
$= \frac{196}{8} = 24.5$

ii. $A.M = \bar{X} = \frac{\sum X}{n} = \frac{200+225+350+375+270+320+290}{7}$
$= \frac{2030}{7} = 290$

4. For each of the data in Q.No.3 Compute arithmetic mean using indirect method.
Solution:
i. Take any constant say 24 and take deviations from it (24)
$A = 24$

X	$D = X - A$
12	$12 - 24 = -12$
14	$17 - 24 = -7$
17	$20 - 24 = -4$
24	$24 - 24 = 0$
29	$29 - 24 = 5$
35	$35 - 24 = 11$
45	$45 - 24 = 21$
$n = 8$	$\sum D = 4$

$\bar{X} = A + \frac{\sum D}{n}$

$= 24 + \frac{4}{8} = 24 + \frac{1}{2} = 24 \times \frac{1}{2} = 24.5$

ii. Take any constant say 270 and take deviations from it (270)

$A = 270$

X	$D = X - A$
200	$200 - 270 = -70$
225	$225 - 270 = -45$
350	$350 - 270 = -80$
375	$375 - 270 = 150$
270	$270 - 270 = 0$
320	$320 - 270 = 50$
290	$290 - 270 = 20$
$n = 7$	$\sum D = 140$

$\bar{X} = A + \frac{\sum D}{n}$

$= 270 + \frac{140}{7} = 270 + 20 = 290$

5. The marks obtained by students of class XI in $mathematics$ are given below. Compare arithmetic mean by direct and indirect methods.

$0 - 90$	2
$10 - 19$	10
$20 - 29$	5
$30 - 39$	9
$40 - 49$	6
$50 - 59$	7
$60 - 69$	1

Solution:

Direct method:

Classes/ Groups	Mid points	f	fx
$0 - 90$	4.5	2	$4.5 \times 2 = 9.0$
$10 - 19$	14.5	10	$14.5 \times 10 = 145.0$
$20 - 29$	24.5	5	$24.5 \times 5 = 122.5$
$30 - 39$	34.5	9	$34.5 \times 9 = 310.5$
$40 - 49$	44.5	6	$44.5 \times 6 = 267.0$
$50 - 59$	54.5	7	$54.5 \times 7 = 381.5$
$60 - 69$	64.5	1	$64.5 \times 1 = 64.5$
		$n = \sum f = 40$	1300

$$\bar{X} = \frac{\sum fx}{\sum f} = \frac{1300}{40} = 32.5$$

Indirect, short cut method

$let\ A = 34.5$

Classes/ Groups	Mid points	f	$D = X - a$	$U = \frac{D}{10}$	fD	$f(U) = -\frac{f(d)}{3}$
$0 - 90$	4.5	2	$4.5 - 34.5 = -30$	-3	-60	-6
$10 - 19$	14.5	10	$14.5 \times 34.5 = -20$	-2	-200	-20
$20 - 29$	24.5	5	$24.5 \times 34.5 = -10$	-1	-50	-5
$30 - 39$	34.5	9	$34.5 \times 34.5 = 0$	0	0	0
$40 - 49$	44.5	6	$44.5 \times 34.5 = 10$	1	60	6
$50 - 59$	54.5	7	$54.5 \times 34.5 = 20$	2	140	14
$60 - 69$	64.5	1	$64.5 \times 34.5 = 30$	3	30	3
Total		$n = \sum f = 40$	1300		-80	-8

$$\bar{X} = h + \frac{\sum fD}{\sum f}$$
$$34.5 + \frac{-80}{40}$$
$$= 34.5 - 2$$
$$= 32.55$$

or

$$\bar{X} = h + \frac{\sum f(U)}{\sum f} \times h$$
$$= 34.5 + \frac{-8}{40} \times h$$
$$= 34.5 + \frac{-8}{40} \times 10$$
$$34.5 - 2 = 32.55$$

6. The following data relates to to ages of children in a school. Compute the mean age by direct and short $-\ cut\ method\ taking\ ant\ provisonal\ mean.$

Class limits	Frequency
$4 - 6$	10
$7 - 9$	20
$10 - 12$	13
$13 - 15$	7
$Total$	50

Also Compute Geometric mean and Harmonic mean.

Solution:

Class limits	Midpoints	Frequency	fx
$4 - 6$	5	10	$5 \times 10 = 50$
$7 - 9$	8	20	$8 \times 20 = 160$
$10 - 12$	11	13	$11 \times 13 = 143$
$13 - 15$	14	7	$14 \times 7 = 98$
$Total$	$n = \sum f = 50$	50	$\sum fx = 451$

$$A.M = \frac{\sum fD}{\sum f} = \frac{451}{50} = 9.02$$

Indirect, short cut method

$$Let\ A = 11$$

Classes/ Groups	f	Midpoint	$D = X - a$	$U = \frac{D}{10}$	fD	$f(U) = -\frac{f(d)}{3}$
$4 - 6$	5	5	$5 - 11 = -6$	-2	-60	-20
$7 - 9$	8	8	$8 - 11 = -3$	-1	-60	0
$10 - 12$	11	11	$11 - 11 = -3$	0	0	7
$13 - 15$	14	14	$14 - 11 = -3$	1	21	-33
Total	$\sum f$				-99	-8

p

$$\bar{X} = A + \frac{\sum fD}{\sum f}$$
$$11 - \frac{99}{50}$$
$$= 11 - 1.98$$
$$= 9.02$$

or $$\bar{X} = A + \frac{\sum f(U)}{\sum f} \times h$$
$$= 11 + \frac{-33}{50} \times 3$$
$$= 11 - \frac{99}{50}$$
$$11 - 1.98 = 9.02$$

Geometric Mean

We proceed as follows:

Class limits	f	Midpoints	$logx$	$flogx$
$4-6$	10	5	0.6987	6.9897
$7-9$	20	8	0.90309	18.0618
$10-12$	13	11	1.04139	13.53807
$13-15$	7	14	1.14613	8.02291
	$\sum f = 50$		$\sum f\ logx$	$= 46.61248$

$$G.M = Antilog\left(\frac{\sum flogx}{\sum f}\right)$$

$$G.M = Antilog\left(\frac{46.61248}{50}\right)$$

$$Antilog(0.9322496) = 8.553$$

Harmonic means:

Class limits	f	Midpoints	$\frac{f}{x}$
$4-6$	10	5	$\frac{10}{5} = 2.0$
$7-9$	20	8	$\frac{20}{8} = 2.5$
$10-12$	13	11	$\frac{13}{11} = 1.18$
$13-15$	7	14	$\frac{7}{14} = 0.50$
	$\sum f = 50$		$\sum f/x = 6.18$

$$H.M = \left(\frac{\sum f}{\sum \frac{f}{x}} = \frac{50}{6.18} = 8.09\right)$$

7. The following data shows the number of children in which in various familiar. Find mode and median.
9,11,4,5,6,8,4,3,7,8,5,5,8,3,4,9,12,8,9,10,6,1,7,11,4,4,8,4,3,2,7,9,10,9,7,6,9,5

Solution:

Writing the observation in Ascending order

2,3,3,3,4,4,4,4,4,4,5,5,5,5,6,6,6,7,7,7,7,7,8,8,8,8,8,9,9,9,9,9,9,10,10,11,11,12

Mode: the most frequent observation $= 9,4$

$Number\ of\ observation = 38$

Therefore, median is the mean of 19th and 20th observation $= \frac{7+7}{2} = 7$

8. Find Model number of heads for the following distributive showing of heads when 5 coins are tossed. Also determine median.

$X(number\ of\ heads)$	Frequency (number of times)
1	3
2	8
3	5
4	3
5	1

Solution:
Mode:
The most frequent observation $= 2$
For median, we make cumulative frequency column.

x	$frequency$	$Cumulative\ frequency$
1	3	3
2	8	3+8=11
3	5	11+5=16
4	3	$16+3=19$
5	1	19+1=20

$$Median = the\ class\ containing\ \left(\frac{n}{2}\right)^{th}\ observation$$
$$= the\ class\ containing\ \left(\frac{20}{2}\right)^{th}\ observation.$$
$$= the\ class\ containing\ (10^{th}) observtaion.$$
$$Median = 2$$

9. The following frequency distribution the weight of boys in kilogram. Compute mean, median, mode.

Class intervals	frequency
$1-3$	2
$4-6$	3
$7-9$	5
$10-12$	4
$13-15$	6
$16-18$	2
$19-21$	1

Solution:

Class intervals	frequency	Mid points(x)	fx	Class Boundaries	Cumulative Frequency
$1-3$	2	2	4		2
$4-6$	3	5	15		2+3
$7-9$	5	8	40		5+5=10
$10-12$	4	11	44		10+4=14
$13-15$	6	14	84		14+6=20
$16-18$	2	17	34		20+2=22
$19-21$	1	20	20		22+1=23
	23		241		

$$Mean = \bar{X} = \frac{\sum fx}{\sum f} = \frac{241}{23} = 10.478$$

Median:

Median class $=$ $class\ containg\ \left(\frac{n}{2}\right)^{th}\ obseravtion.$

$= \left(\frac{23}{2}\right)^{th} = (11.5)^{th}$observation

Median class is $9.5 - 12.5$

Here $l = 9.5, c = 10, f = 4, h = 3$

Median= $l + \frac{h}{f}\left(\frac{n}{c} - c\right)$

$$= 9.5 + \frac{3}{4}\left(\frac{23}{2} - 10\right) = 9.5 + \frac{3}{4}\left(\frac{3}{2}\right) = 9.5 + \frac{9}{8} = 9.5 + 1.125 = 10.625$$

Mode: $Mode = l + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times h$

Here$l = 12.5, f_m = 6, f_1 = 4, f_2, h = 3$

$$\because Mode = 12.5 + \frac{6-4}{2(6)-4-2} \times 3 = 12.5 + \frac{2}{6} \times 3 = 12.5 + 1 = 13.5$$

10. A student obtained the following marks at a certain examination: English 73, Urdu 82, Mathematics 80, History 67 and Science 62.

i. If the Wight accorded these marks are 4,3,3,4 $and\ 2.repectively.$ what is an appropriate average marks?

ii. What is the average mark if equal weights are used?

Solution:

Marks(x)	Weight(w)	xw
73	4	$73 \times 4 = 292$
82	3	$82 \times 3 = 246$
80	3	$80 \times 3 = 240$
67	2	$67 \times 2 = 134$
62	2	$62 \times 2 = 124$
$\sum x = 364$	$\sum w = 14$	$\sum xw = 1036$

$$(i) \qquad \bar{X}_n = \frac{\sum Xw}{\sum w} = \frac{1036}{14} = 74$$

$$(ii) \qquad \bar{X} = \frac{\sum x}{n} = \frac{364}{5} = 72.8$$

11. On a vacation trip a family bought 21.3liters of petrol at 39.90 rupees per liter, 18.7 liters at 42.90 rupees per liter, and 23.5 liters at 40.90 rupees per liter find the mean price paid per liter.

Solution:

X	W	XW
21.3	39.90	$(21.3)(39.90) = 849.87$
18.7	42.90	$(21.3)(39.90) = 849.87$
23.5	40.90	$(21.3)(39.90) = 849.87$
$\sum x = 63.5$		$\sum xW = 2613.25$

Mean price=$\frac{\sum XW}{\sum X} = \frac{2613.25}{63.5} = 41.15\ rupees\ per\ liter$

12. Calculator simple moving average of 3 years from the following data;

Years	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**
Valves	**102**	**108**	**130**	**140**	**1158**	**180**	**196**	**210**	**220**	**230**

Solution:

Years	Values	3-years moving total	3- years moving average
2001	102	-	-
2002	108	**340**	**340/3=113.33**

2003	130	**378**	**378/3=126.00**
2004	140	**428**	**428/3=142.67**
2005	158	**478**	$\frac{478}{3} = 159.33$
2006	180	**534**	**534/3=178.00**
2007	196	**586**	**586/3=195.33**
2008	210	**626**	**626/3=208.67**
2009	220	**660**	**660/3=220.00**
2010	230	**-**	

13. Determine graphically for the following data and check your answer by using formulae.

- **i. Median and Quartiles using cumulative frequency polygon.**
- **ii. Mode using Histogram**

Class Boundaries	Frequency
$10 - 20$	2
$20 - 30$	5
$30 - 40$	9
$40 - 50$	6
$50 - 60$	4
$60 - 70$	1

Solution:

Class Boundaries	Frequency	$c.f$
$10 - 20$	2	2
$20 - 30$	5	7
$30 - 40$	9	16
$40 - 50$	6	22
$50 - 60$	4	26
$60 - 70$	1	27

Median Class $Q_3 Clas$

Median Class $= \left(\frac{n}{2}\right)^{th}$ $observation = \left(\frac{27}{2}\right)^{th} = (13.5)^{th} observation.$

Median= $l + \frac{h}{f}\left(\frac{n}{2} - c\right)$

Here $l = 30, h = 10, f = 9, n = 2.7. c = 7$

Thus median $x = 30 + \frac{10}{9}\left(\frac{27}{2} - 7\right) = 30 + \frac{10}{9}\left(\frac{13}{2}\right) = 30 + 7.22 = 37.22$

To find Q_1

We have to find 3 $\left(\frac{n}{4}\right)^{th}$ $observation.$

Next Not Solved. Not important according to exam

i. Range:

Range measure the extent of variation between two extreme observations of a data set.

It is given by the formula:

$X_{max} - X_{min} = X_m - X_o$

Where $X_{max} = X_m = the\ maximum, highest\ or\ largest\ observation.$

$$X_{min} = X_o = the\ minimum\ lowest\ or\ smallest\ observation.$$

The formula to find range for grouped continuous data us given below.

Range $= (Upper\ class\ boundary\ of\ last\ group) - (Lower$ **class boundary of first group).**

i. Variance:

Variance is defined as the mean of the squared deviation of $x_i(i = 1, 2, 3, \dots, n)$ observation from their arithmetic mean. In symbols,

$$Variance\ of\ X = Var(X) = S^2 = \frac{\sum(X - \bar{X})^2}{n}$$

ii. **Standard Deviation**

Standard deviation is defined as the positive square root of mean of the squared deviations of $X_i(i = 1, 2, 3, \dots, n)$ observations from their arithmetic mean. In symbols we write

$$standard\ Devaition\ of\ X = S.D(X) = S = \sqrt{\frac{\sum(X - \bar{X})^2}{n}}$$

Computations of Variance and Standard Devotions

We uses the following to compute Variance and standard Deviations for Ungrouped and Grouped Data.

Ungrouped Data:

The formula of Variance is given by

$$Var(X) = S^2 = \frac{\sum X^2}{n} - \left(\frac{\sum X}{n}\right)^2$$

And standard Deviation

$$S.D(X) = S = \sqrt{\left[\frac{\sum X^2}{n} - \left(\frac{\sum X}{n}\right)^2\right]}$$

Exercise 6.3

1. What do you understand by Dispersion?
   Dispersion means the spread or scatter ness of observations in a data set. By dispersion means the extent to which observations in a sample or n a population are spread out. The main measure of dispersion are range, variance and standard deviation's.
2. How do you define measure of dispersion?
   The measure that are used to determine the degree or extent of variation in a data set are called measure of dispersion.
3. Define Range, Standard deviation and Variance.

Solution:

ii. **Range:**

Range measure the extent of variation between two extreme observations of a data set.

It is given by the formula:

$$X_{max} - X_{min} = X_m - X_o$$

Where $X_{max} = X_m = the\ maximum, highest\ or\ largest\ observation.$

$$X_{min} = X_o = the\ minimum\ lowest\ or\ smallest\ observation.$$

The formula to find range for grouped continuous data us given below.

Range $= (Upper\ class\ boundary\ of\ last\ group) - (Lower$ **class boundary of first group).**

iii. **Variance:**

Variance is defined as the mean of the squared deviation of $x_i(i = 1, 2, 3, \dots, n)$ observation from their arithmetic mean. In symbols,

$$Variance\ of\ X = Var(X) = S^2 = \frac{\sum(X - \bar{X})^2}{n}$$

iv. **Standard Deviation**

Standard deviation is defined as the positive square root of mean of the squared deviations of $X_i(i = 1, 2, 3, \ldots, n)$ observations from their arithmetic mean. In symbols we write

$$standard\ Devaition\ of\ X = S.D(X) = S = \sqrt{\frac{\sum(X-\bar{X})^2}{n}}$$

Computations of Variance and Standard Devotions

We uses the following to compute Variance and standard Deviations for Ungrouped and Grouped Data.

Ungrouped Data:

The formula of Variance is given by

$$Var(X) = S^2 = \frac{\sum X^2}{n} - \left(\frac{\sum X}{n}\right)^2$$

And standard Deviation

$$S.D(X) = S = \sqrt{\left[\frac{\sum X^2}{n} - \left(\frac{\sum X}{n}\right)^2\right]}$$

4. The salaries of five teachers in Rupees are as follows.
11500,12400,15000,14500,14800.
find Range and Standard devitions

Solution:

$X = 11500, 12400, 15000, 14500, 14800.$

Here $X_{min} = 11500$, $X_{max} = 15000$

$$Range = X_{max} - X_{min}$$
$$= 15000 - 11500$$
$$= 3500$$
$$\bar{X} = \frac{\sum x}{n}$$
$$= \frac{11500 + 12400 + 15000 + 14500 + 14800}{5}$$
$$= \frac{68200}{5} = 13640$$

X	$X-\bar{X}$	$(X-\bar{X})^2$
11500	-2140	4579600
12400	-1240	1537600
15000	1360	1849600
14500	860	739600
14800	1160	1345600

$$\sum(X-\bar{X})^2 = 10052000, \qquad n = 5$$

$$S.D(X) = S = \sqrt{\left[\frac{\sum X^2}{n} - \left(\frac{\sum X}{n}\right)^2\right]}$$
$$= \sqrt{\frac{10052000}{5}}$$
$$= \sqrt{2010400}$$
$$= 1417.88$$

5. (a) Find the standard deviation "S" of each set of numbers:

i. 12,6,7,3,15,10,18,5

ii. 9,3,8,8,9,8,9,18.

(b) Calculate variance for the data 10,8,9,7,5,12,8,6,8,2

Solution:

i.

X	$X-\bar{X}$	$(X-\bar{X})^2$
12	2.5	6.25
6	-3.5	12.25
7	-2.5	6.25
3	-6.5	42.25
15	5.5	30.25
10	0.5	0.25
18	8.5	72.25
5	-4.5	20.25

$$\sum X = 76 \qquad \sum (X-\bar{X})^2 = 190, n = 8$$

$$\bar{X} = \frac{76}{8} = 9.5$$

$$\boldsymbol{S.D(X) = S} = \sqrt{\left[\frac{\sum X^2}{\boldsymbol{n}} - \left(\frac{\sum X}{\boldsymbol{n}}\right)^{\boldsymbol{2}}\right]}$$

$$= \sqrt{\frac{190}{8}}$$

$$= \sqrt{23.75}$$

$$= 4.87$$

ii.

X	$X-\bar{X}$	$(X-\bar{X})^2$
9	0	0
3	-6	36
8	-1	1
8	-1	1
9	0	0
8	-1	1
9	0	0
18	9	81

$$\sum X = 72 \qquad \sum (X-\bar{X})^2 = 120, n = 8$$

$$\bar{X} = \frac{\sum X}{n} = \frac{7o}{8} = 9$$

$$\boldsymbol{S.D(X) = S} = \sqrt{\left[\frac{\sum X^2}{\boldsymbol{n}} - \left(\frac{\sum X}{\boldsymbol{n}}\right)^{\boldsymbol{2}}\right]}$$

$$= \sqrt{\frac{120}{8}}$$

$$= \sqrt{15} = 3.87$$

(b) Calculate variance for the data 10,8,9,7,5,12,8,6,8,2

Solution:

X	$X-\bar{X}$	$(X-\bar{X})^2$
10	2.5	6.25
8	0.5	25
9	1.5	2.25
7	-0.5	.25
5	-2.5	6.25
12	4.5	20.25
8	0.5	.25

6	-1.5	2.25
8	0.5	.25
2	-5.5	30.25

$$\sum X = 75 \qquad \sum (X-\bar{X})^2 = 68.5, n = 10$$

$$\bar{X} = \frac{\sum X}{n} = \frac{75}{10} = 7.5$$

$$\boldsymbol{Variance\ of\ X = Var(X) = S^2} = \frac{\sum (X-\bar{X})^2}{\boldsymbol{n}}$$

$$= \frac{68.5}{10} = 6.85$$

6. The length of 32 items are given below. Find the mean length and standard deviation of the distribution.

Length	$20-22$	$23-25$	$26-28$	$29-31$	$32-34$
frequency	3	6	12	9	2

Solution:

$C.I$	f	Mid points(x)	fx	$X-\bar{X}$	$(X-\bar{X})^2$	$f(X-\bar{X})^2$
$20-22$	3	21	63	-6	36	108
$23-25$	6	24	144	-3	9	54
$26-28$	12	27	324	0	0	0
$29-31$	9	30	270	3	9	81
$32-34$	2	33	66	6	36	72
total	32		$\sum fx = 867$		90	315

$\bar{X} = \frac{\sum fx}{n} = \frac{867}{32} = 27.093 = 27approx$ $\qquad \bar{X} = \frac{\sum X}{n} = \frac{75}{10} = 7.5$

$$\boldsymbol{S.D(X) = S} = \sqrt{\left[\frac{\sum X^2}{\boldsymbol{n}} - \left(\frac{\sum X}{\boldsymbol{n}}\right)^{\boldsymbol{2}}\right]} = \sqrt{\frac{315}{\boldsymbol{32}}} = \sqrt{9.84375} = \boldsymbol{3.3137}$$

7. For the following distribution of marks calculator Range

	Frequency/No.
$33-40$	28
$41-50$	31
$51-60$	12
$61-70$	9
$71-75$	5

Solution:

$C.I$	Class Boundaries	f
$33-40$	$32.5-40.5$	28
$41-50$	$40.5-50.5$	32
$51-60$	$50.5-60.5$	12
$61-70$	$60.5-70.5$	9
$71-75$	$70.5-75.5$	5

Here

$$X_{max} = 75.5$$
$$X_{min} = 32.5$$
$$Range = X_{max} - X_{min}$$
$$= 75.5 - 32.5 = 43$$